Understanding and Solving Mechanical Instabilities

Paul A. Zaffiro, Procter & Gamble Company, Cincinnati, Ohio

Mechanical instabilities are self-excited vibration phenomena that occur in the machine tool, manufacturing, and process industries. Though not common, instabilities can lead to destructive vibration levels and costly repairs. This article reviews the characteristics of unstable vibration phenomena, and explains the cause of instabilities. It compares instabilities to resonant excitation problems to highlight the similarities and differences. It then presents methods for solving instability problems, along with two examples from the author's experience.

Most engineers and technicians are aware of the Tacoma Narrows bridge disaster. This suspension bridge failed catastrophically in November 1940, five months after completion, due to high amplitude oscillations caused by the wind! Though most believe this disaster was due to resonant excitation, the root cause was a mechanical instability.

Mechanical instabilities are a class of self-excited vibration phenomena.* Self-excited vibration problems begin to vibrate of their own accord spontaneously. The distinguishing feature of mechanical instabilities is the presence of a feedback mechanism between the structural vibration and the oscillation of some key component of the process or structure. When this "feedback loop" becomes unstable, the vibration amplitude increases with time, sometimes to destructive levels.

For the Tacoma Narrows bridge, an unstable feedback loop existed between the bridge lateral (i.e., crosswind) vibration and the aerodynamic forces exerted on the bridge by the wind. That is, the aerodynamic forces varied as a function of the bridge vibration and vice versa. As the vibration velocity grew in amplitude, so did the aerodynamic forces. The feedback loop was unstable in that the aerodynamic forces and the bridge oscillation reinforced one another over time. Finally, the amplitude of oscillation became so large that the center section of the bridge disintegrated.

Instabilities, while relatively rare compared to resonance, must be understood by the practicing vibration specialist. These problems give rise to potentially destructive levels of vibration. The purposes of this article are the following: (1) to present the characteristics of mechanical instabilities so that they can be identified in the field; (2) to explain the nature and causes of instabilities; and (3) to present practical methods for solving instabilities. The article concludes with two examples.

How to Identify an Instability

Mechanical instabilities are identified through careful analysis of the operating vibration characteristics of a structure in conjunction with knowledge of the structural dynamics. Mechanical instabilities generate the following unusual vibration characteristics:

Intermittent High Vibration. Instabilities generate episodes of very high vibration relative to "normal" levels. In the unstable regime, operating vibration can be hundreds of times higher than in the stable regime. It often appears as if the vibration is either 'on' or 'off.'

Absence of a Periodic Excitation Source. Since instabilities are self-generated, they often develop in the absence of a periodic vibration source. The alternating forces which sustain the instability are generated by the instability itself. External forces are not required. The absence of a periodic excitation source is an indicator that the problem may be an instability.

Substantial Vibration Amplitude Variation With Time. Instabilities often give rise to large variations in vibration amplitude. Vibration amplitudes can increase or decrease by an order of magnitude relatively quickly. For example, vibration in one system grew from 0.05 ips-pk to 3 ips-pk in less than 5 sec when operating in an unstable regime.

Nonsynchronous Pure Tone Vibration Frequency. Instabilities often occur at frequencies which are not integer multiples of speeds of the rotating elements in the system. While this is not always true (calender barring for example), non-synchronous vibration may be an indicator that the problem is not resonant excitation.

Vibration Sensitive to Speed, Loading, or Other Process Parameters. Instabilities are extremely sensitive to process conditions, including machine speed. Instabilities can be initiated or terminated with a change of only a few percent in a process variable.

Dominant Vibration Frequency Near a Natural Frequency of the Structure. Instabilities are often associated with structural natural frequencies. This is because instabilities are most likely to occur where the structure is dynamically compliant to the primary forcing function in the system. Since natural frequencies cause dynamic compliance, instabilities develop at nearby frequencies.

Presence of a Mechanical Feedback Mechanism. Instabilities require feedback. All mechanical instabilities have an inherent feedback mechanism between the vibration of the structure and variation in some key component of the process. Sometimes the mechanism will not be immediately apparent, and will require in-depth investigation of the process and equipment. If such a mechanism is present, an instability should be strongly suspected.

Several of the above characteristics are unique to instabilities, such as the presence of a feedback mechanism and the presence of high pure-tone vibration in the absence of an excitation source. However, some characteristics are equally true of resonance. In the next section, we'll compare these two phenomena, and find that they are vastly different.

Instabilities vs. Resonance

We saw in the last section that instabilities and resonance share some common characteristics. For example, both are sensitive to process conditions (i.e., speed) and both occur in the vicinity of structural natural frequencies. Despite these similarities, instabilities and resonance are vastly different phenomena. In this section, we'll compare and contrast these two phenomena.

Resonance is high vibration resulting from operation at or near a natural frequency of a structure. It is a forced vibration problem and can be represented by the block diagram shown in Figure 1.

The amplitude of vibration is directly proportional to the amplitude of the excitation force, assuming the system is linear. Thus, if the force doubles, the amplitude of vibration doubles. Similarly, the amplitude of vibration is directly proportional to the magnitude of the compliance function at the excitation frequency. If the compliance of the system doubles (i.e., the system stiffness is reduced by half), the amplitude of

*Other classes of self-excited vibration problems include aeromechanical (flutter), aerodynamics (stall, separation, musical instruments), aerothermodynamics (flame instability), and feedback networks (electromechanical, hydraulic, pneumatic).

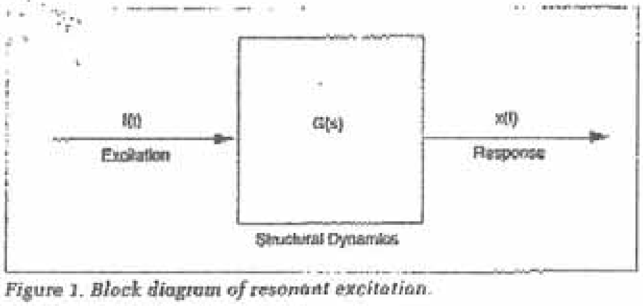

Figure 1. Block diagram of resonant excitation.

vibration doubles.

By contrast, while instabilities are often associated with natural frequencies, they are not caused by them. The root cause of an instability is the presence of a feedback mechanism which sets up an unstable feedback loop within the system. Instabilities will always contain feedback mechanisms. They can be represented by the block diagram shown in Figure 2.

For instabilities, the amplitude of vibration is primarily a function of the stability of the system. The amplitude of vibration is not necessarily impacted by the presence or magnitude of the excitation force, or the compliance of the structure. The issue is whether the system is stable. Vibration amplitude can be hundreds of times higher in the unstable regime than the stable regime.

Resonance is a forced-vibration problem. Resonance requires an alternating force with significant energy near the natural frequency of the structure. The force is independent of the motion and persists when the motion is stopped. Resonance can be eliminated if the frequencies of the excitation source and the natural frequency can be sufficiently separated.

Instabilities are self-excited. The alternating force that sustains the motion is created and controlled by the motion itself. When the motion stops, the force disappears. Instabilities require energy, but the frequency content of the energy is not significant. Reducing or eliminating excitation sources will have little impact on the problem, since instabilities are not caused by external alternating forces.

Table 1 summarizes the similarities and differences between resonance and instabilities.

Controlling Mechanical Instabilities

Controlling (or eliminating) instabilities requires making the feedback loop stable. In this section, we'll develop a simple dynamic model for a common mechanical instability: machine tool chatter in a turning operation (e.g., a lathe). We'll use this model to learn the following: (1) that instabilities involve positive feedback loops that make the system unstable; and (2) that three methods are available for stabilizing these systems.

Model Development

Single-point cutting, a class of machine tool chatter problems, has been extensively studied and modeled using the feedback paradigm. Approaching chatter as a feedback loop was first done by Merritt.[1] His model accurately predicted the stability envelope for turning operations. Later researchers enhanced the theory, but continued to approach the problem using the feedback paradigm. In the following paragraphs, we'll develop the model and see that it includes feedback loops.

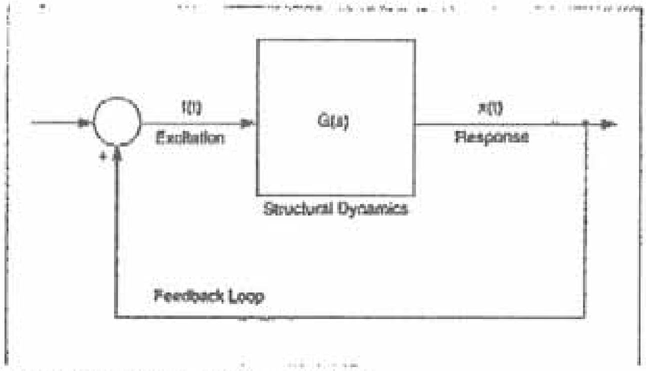

Figure 2. Block diagram of an instability.

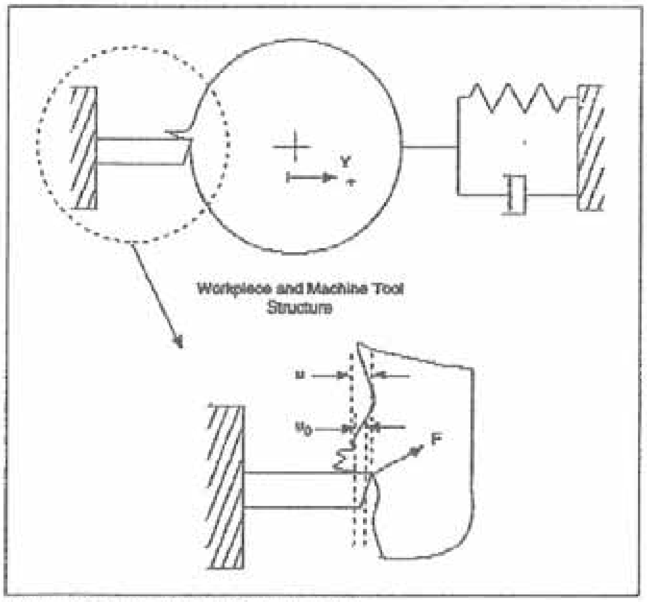

Figure 3. Single point cutting operation.

Following Merritt's development of the chatter loop, a single-point tool performing orthogonal cutting is shown in Figure 3. Assume that the tool is rigidly mounted. The feed rate is adjusted to obtain an average, or steady-state depth of cut, $u_0(t)$. In the steady-state condition, the structure maintains a certain deflection, $y(t)$, caused by the steady-state cutting force. All equations will be written about this point of equilibrium.

Referring to Figure 3, the instantaneous depth of cut, $u(t)$, is *decreased* as the structure (workpiece) moves *away* from the tool, that is, as $y(t)$ increases. If this occurs, a lump of material will be left on the workpiece. This lump increases the uncut chip thickness one revolution, or T seconds later. Thus, the uncut chip thickness can be written as:

$$u(t) = u_0(t) - y(t) + y(t-T) \tag{1}$$

Note in particular that the uncut chip thickness is a function of the position of the workpiece. *This is the feedback tie.*

The instantaneous uncut chip thickness, along with the

Table 1. Comparison of resonance and instabilities.

Item	Resonance	Instabilities
Vibration Amplitude	Function of Excitation	Function of System Stability
Relationship to Structural Natural Frequencies	Primary Cause	Associated With
Excitation Source	Required	Not Required
Presence of Feedback Mechanism	No	Yes
Vibration Sensitive to Process Parameters*	Typically No	Yes
Vibration Amplitude a Function of Frequency	Yes	Possibly. Function of Instability Severity
Dominant Vibration Frequency	Synchronous Pure Tone	Typically Nonsynchronous Pure Tone
Vibration Amplitude Varies Significantly With Time	No	Yes

*Other than equipment speed

mechanical properties of the workpiece material, determine the resultant cutting force exerted on the system. Under steady-state conditions, the machining process can be represented as a single stiffness, k_c, called the cutting stiffness. The cutting stiffness is directly proportional to the width of cut, the workpiece material, and the tool geometry. The cutting force, $F(t)$, can be given as:

$$F(t) = k_c \cdot u(t) \tag{2}$$

The cutting force acts to displace the workpiece and structure of the machine. Therefore, the dynamic compliance of the structure is critical to the chatter problem. Assuming that the structural dynamics can be represented adequately as a lumped-parameter model with one degree of freedom, and that the cutting force is applied in the direction of machine motion, we have:

$$F(t) = m\ddot{y} + c\dot{y} + k_m y \tag{3}$$

where m, c, and k_m are the lumped mass, damping, and stiffness of the structure.

This equation can be transformed into the Laplace domain and rearranged to give:

$$y(s) = \frac{1}{k_m} \cdot G_m(s) \tag{4}$$

where

$$G_m(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n + \omega_n^2} \tag{5}$$

$G_m(s)$ is the normalized dynamic compliance of the structure and $\omega_n^2 = k_m/m$ and $\xi = c/[2(k_m m)^{1/2}]$.

Equation (4) is the relative dynamic compliance between the tool and workpiece. In most structures, including machine tools, this function is determined experimentally.

Equations (1), (2), and (4) define the dynamics of the system. These equations can be combined to create the block diagram shown in Figure 4. The feedback tie is provided by the interdependence between the uncut chip thickness, $u(t)$, and the structural vibration, $y(t)$. When the loop is unstable, variation in the uncut chip thickness generates alternating cutting forces which cause the structure to vibrate. As the structure vibrates, the uncut chip thickness variation increases, causing greater alternating cutting forces, which cause greater oscillation of the structure, and so on. This unstable condition is called regenerative machine tool chatter. Chatter continues to build until limited by nonlinear response.

Making the Chatter Loop Stable

Since mechanical instabilities involve feedback mechanisms, feedback control theory provides a suitable framework from which to study their stability. However, a rigorous mathematical treatment of system stability is outside the scope of this article. The approach taken here will be practical and heuristic rather than mathematical. The objective is to gain insight into solving instabilities.

First, note the presence of two feedback loops in Figure 4. The inner loop, or primary feedback path, provides *negative* feedback. Negative feedback increases system stability. The outer loop, or regenerative path, provides *positive* feedback. Positive feedback tends to destabilize the system and allows for the onset of unstable vibration behavior. All instabilities have inherent feedback loops similar to those depicted in Figure 4.

Stabilizing these systems requires either eliminating the feedback loops or modifying the system such that the positive, reinforcing (i.e., regenerative feedback) does not occur. The following are three methods for eliminating or controlling instabilities:

1. Eliminating the Feedback Path. Eliminating the feedback path(s) precludes the possibility of regenerative feedback. It decouples the structural vibration from variations in process parameters, and prevents the onset of an instability.

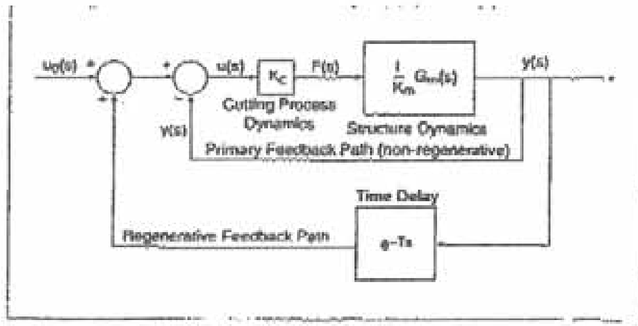

Figure 4. Block diagram of chatter loop.

In some systems, this approach is not feasible. For example, in the machining operation above, eliminating the feedback path would mean that the workpiece and tool can not come in contact. Clearly, this is impractical for conventional machining.

To apply this approach to machining, non-contact machining methods must be employed. Water-jet, EDM, and laser machining are just three examples of chatter-free, non-contact machining methods.

In other systems, the key process quantity which provides the feedback tie must be identified and studied to determine whether the tie can be broken. Once the tie is identified, potential modifications to the process can be considered.

2. Reducing the "Cutting Stiffness." "Softer" workpiece materials (e.g., aluminum) are less likely to chatter than "harder" materials (e.g., 4140 alloy steel). Let's look at the block diagram of the chatter loop (Figure 4) to see why.

The hardness of the workpiece material is represented by the "cutting stiffness" block, k_c. Softer materials have a lower value of k_c than harder materials. For the same depth and width of cut, a softer material requires less cutting force than a harder material. Less cutting force generates less structural vibration. Less structural vibration means that the tendency for regenerative feedback is reduced, and thus the uncut chip thickness variation is reduced. Clearly, softer workpiece materials, which have lower "cutting stiffnesses," move the system toward stability.

In other systems, the "cutting stiffness" represents the feedback tie that converts structural vibration into an alternating force that acts on the structure. For the Tacoma Narrows bridge, the "cutting stiffness" was the effect of the shape of the bridge on the aerodynamic forces acting on the bridge. This effect could have been reduced by changing the bridge geometry. The case histories in the next section provide other examples of the "cutting stiffness" and ways to reduce it.

3. Increasing the Dynamic Stiffness of the Structure. Increasing the dynamic stiffness of the machine tool structure is an effective means of reducing chatter. To this end, the use of steadies, rigid tools and clamps, and the optimum positioning of machine tool axes have been used to good effect. The use of tuned mass absorbers and other damping devices have also met with good success.[2]

The effectiveness of increasing the dynamic stiffness can be seen in Figure 4. Increasing k_m and reducing $G_m(s)$, both of which increase the dynamic stiffness, tend to reduce the vibration of the structure for a given alternating input force. If k_m were infinite, the structural vibration would be zero, and the feedback loops would be open. Thus, increasing the dynamic stiffness tends to stabilize the loop by reducing feedback to the process.

Increasing the dynamic stiffness is perhaps the most practical means of controlling instabilities. Dynamic stiffness is a function of the mass, static stiffness, and damping in the structure. While increasing the static stiffness may be helpful, the overall impact on the dynamic stiffness must be evaluated. Often, adding damping is a more effective and practical means of increasing the dynamic stiffness.

The balance of this article will detail two instabilities and

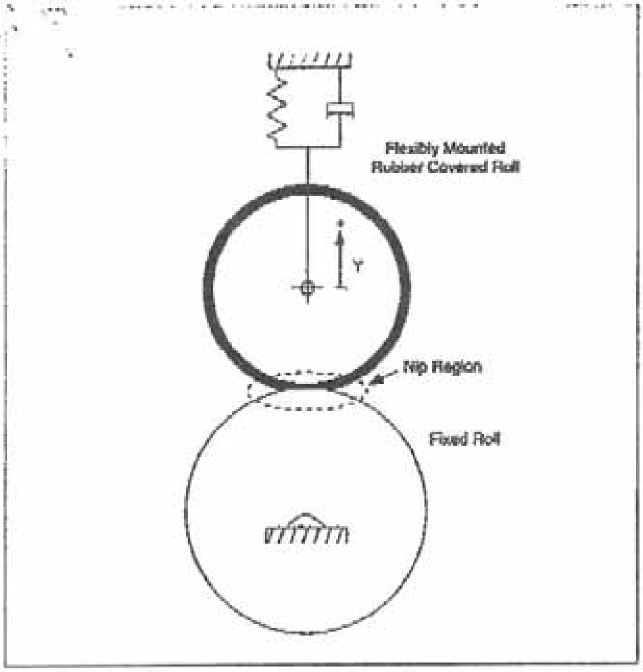

Figure 5. Dynamics of two-roll stack.

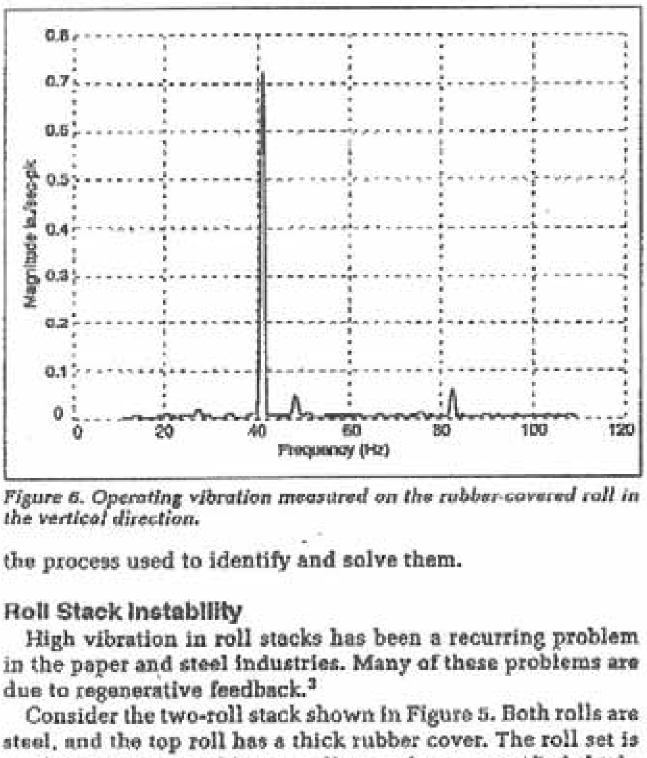

Figure 6. Operating vibration measured on the rubber-covered roll in the vertical direction.

the process used to identify and solve them.

Roll Stack Instability

High vibration in roll stacks has been a recurring problem in the paper and steel industries. Many of these problems are due to regenerative feedback.[3]

Consider the two-roll stack shown in Figure 5. Both rolls are steel, and the top roll has a thick rubber cover. The roll set is used to compress a thin, compliant web to a specified thickness in a high-speed process.

A stack from one particular process had the following characteristics:

- Normal running vibration below 0.05 ips-pk.
- Episodes of high amplitude vibration (up to 0.75 ips-pk) at 41.25 Hz (Figure 6).
- Vibration at an integer multiple of the rotation rate of the rubber-covered roll.
- Out of phase motion of the rolls in the vertical direction (i.e., bouncing of the rolls).
- Amplitude variation from 0.05 ips-pk to 0.75 ips-pk over a relatively short time period (20 min).
- High sensitivity to machine speed.
- Proximity to a major structural resonance of the roll set at 43 Hz.

Analysis of the operating and nonoperating characteristics of the stack was used to identify the instability. Key indicators were the intermittent nature of the vibration, the variation in amplitude with time, and the proximity of a major system resonance. Further, no source of excitation at 41.25 Hz was found. However, unlike most instabilities, the vibration frequency was synchronous with a harmonic of the roll rotation rate.

The next task was to identify the feedback mechanism. The roll stack instability was associated with feedback between the relative vibration of the rolls and a compressive set pattern in the surface of the rubber-covered roll. At certain speeds and loads, this feedback loop became unstable, resulting in regenerative feedback.

In a regenerative feedback scenario, variation in the compressive set in the rubber cover (due to deformation in the nip) generates alternating separation forces between the rolls. The alternating separation forces cause the rolls to vibrate out of phase (due to the proximity of the structural resonance). As the roll stack vibrates, the compressive set pattern is reinforced, generating greater alternating separation forces, which cause more oscillation of the roll stack, and so on. The vibration continues to build until limited by nonlinear response.

Unstable vibration occurs only when an integer multiple of the roll rotation rate is close to the resonance frequency associated with the instability. An integer number of hills/valleys is necessary for the compressive set pattern to reinforce itself with each successive roll rotation. When the speed is such that the frequency of vibration (number of hills/valleys × roll rate) is near the resonant frequency of the roll set, the stack vibrates due to its dynamic compliance. The lack of dynamic stiffness promotes the onset of the instability.

A block diagram model similar to the chatter model (Figure 4) could be developed. The "cutting stiffness" block becomes the roll cover stiffness. The structural dynamics block contains the relative dynamics of the two rolls. The regenerative feedback path would include a time delay associated with one roll revolution as well as a term describing the relaxation of the rubber cover. Such a model has been used to simulate roll stack instabilities with some success.

Two solutions were considered: (1) controlling the instability by controlling process conditions; and (2) modifying the dynamics of the stack to eliminate the instability.

Controlling the instability through process conditions was partially successful initially. The stability of the stack was sensitive to nip load and speed. Higher nip loads increased the compressive set in the rubber cover and stiffened the portion of the cover in the nip, thus increasing the "cutting stiffness" and reducing the system stability. A corrective measure was to run the stack with lower nip loads.

The rotational speed of the rolls also impacted the stability of the stack. The instability was most likely to occur when harmonics of the rubber-covered roll were near the bounce mode of the roll set. Thus, if vibration began trending up, the speed of the stack (and the process) was changed. This approach was eventually abandoned because it proved to be impractical for the wide range of speeds and loads that the process required.

Modifying the stack to eliminate the instability was deemed to be a more robust solution. Each of the three options mentioned earlier was considered. Eliminating the feedback loop was impossible given the physics of the process. Decreasing the "cutting stiffness," which involved using a softer rubber cover, would have negatively impacted the process. Further, the resulting decrease in the system natural frequency may have been detrimental. Increasing the dynamic stiffness was the only feasible alternative for this process.

Several options for increasing the dynamic stiffness were considered. As with all mechanical systems, the dynamic stiffness is controlled by the mass, static stiffness, and damping of the roll stack. While increasing static stiffness and mass increase the dynamic stiffness (and the resonance frequency), these approaches would have required substantial modification of the stack structure. Additional damping could be added more easily and would provide substantial benefit.

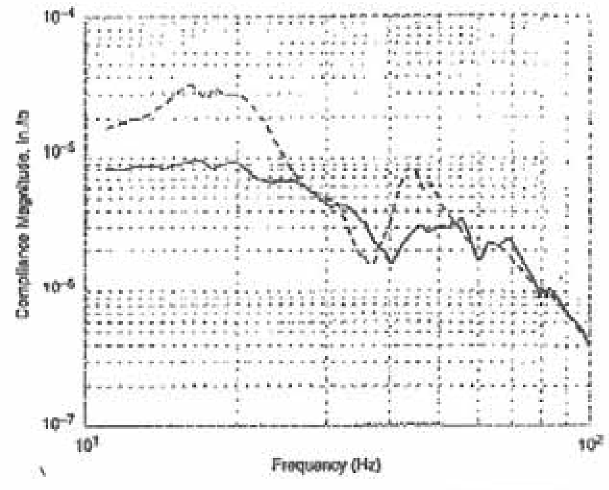

Figure 7. Driving point frequency response of the rubber-covered roll in the vertical direction with dampers (solid) and without dampers (dashed).

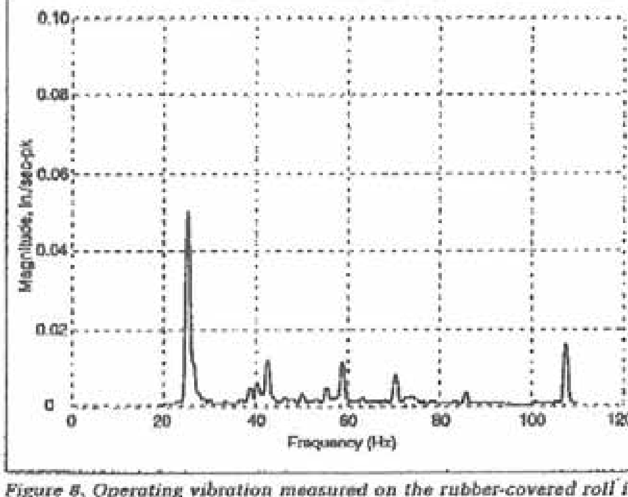

Figure 8. Operating vibration measured on the rubber-covered roll in the vertical direction after damper engagement.

The addition of damping eliminated the instability. The dynamic stiffness of the roll set was increased by a factor of three, as shown in Figure 7. Vibration was reduced by a factor of fifty. An operating spectrum taken following damper engagement is shown in Figure 8.

The factor of fifty reduction in vibration is not unusual when an instability is eliminated. Instabilities, by nature, generate large amplitudes due to positive, reinforcing feedback. When the loop is stabilized, vibration amplitudes return to "normal" levels.

Finally, note that the reduction in vibration is not proportional to the increase in dynamic stiffness. In this example, the dynamic stiffness was increased by a factor of only three, while the vibration was reduced by a factor of fifty! The issue with instabilities is system stability, not system stiffness.

Mixing Tank Instability

Figure 9 depicts a sectional view of a large mixing tank. Attached to the center shaft are several arms which terminate in scrapers. There is a small gap between the scrapers and the shell of the mixer. The shaft rotates at a relatively slow speed. In operation, material is fed into the mixer, creating a relatively stiff coating on the interior of the shell. This coating is machined by the scrapers, which creates the final product.

High vibration was noted immediately following start-up of

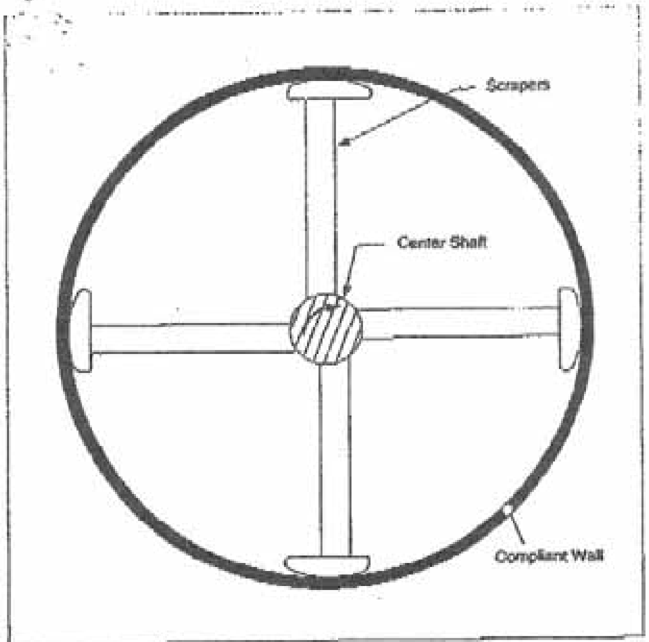

Figure 9. Cross section of mixing tank.

the mixer. A typical time history and corresponding spectrum are shown in Figure 10. A modal analysis was performed on the shaft and shell to determine the dynamics of the machine. The driving point FRF on the shaft in the vertical direction is shown in Figure 11.

The unique character of the vibration time history (Figure 10) indicated that this was not a typical rotating equipment problem. Notice how the vibration builds very quickly to a maximum and then falls off abruptly. These bursts of energy

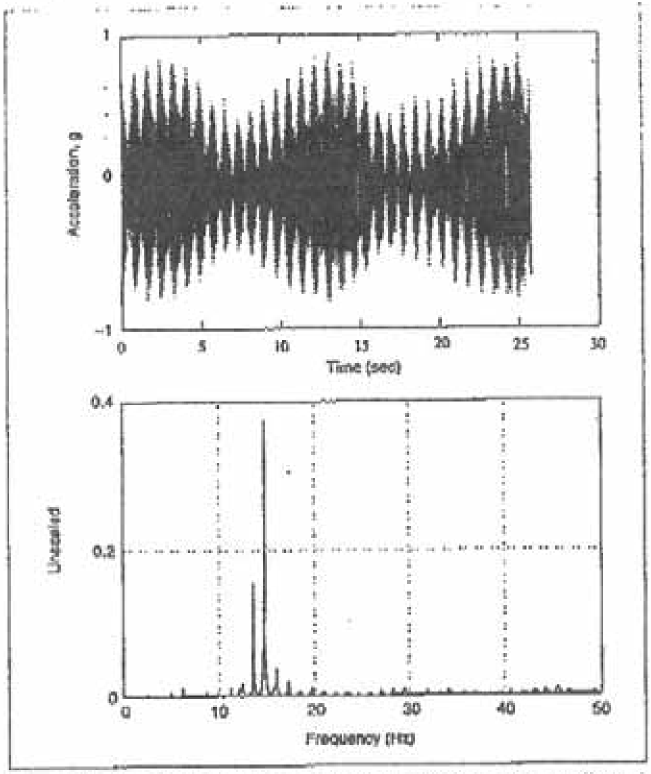

Figure 10. Time history (upper) and corresponding spectrum (lower) measured on mixer frame during unstable operation.

occurred every 15 to 60 sec depending on process conditions. Amplitudes often exceeded 5 ips-pk.

Further investigation revealed that the dominant vibration

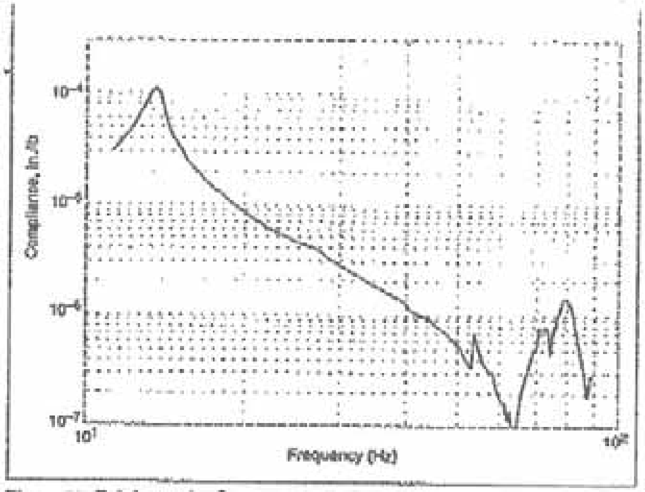

Figure 11. Driving-point frequency response function measured on mixer center shaft in the vertical direction.

frequency was a fractional multiple of shaft RPM. The mode shape revealed that the shaft natural frequencies were in the range of the dominant operating vibration frequencies (compare Figures 10 and 11). Finally, it was noted that high vibration only occurred when the product coating was present on the interior of the shell.

After two months of testing under a variety of process conditions, it was concluded that the mixer vibration was due to a mechanical instability. Once the process was understood, the feedback mechanism was identified. Feedback existed between the coating on the shell and the shaft vibration. Once perturbed, the shaft vibration caused an oscillatory pattern to be machined in the shell coating. Thickness variations in the coating generated alternating forces which increased the shaft vibration. The feedback process continued, increasing the vibration until the forces became so large that the material coating could not sustain the pattern. At this point, the scrapers stripped the pattern away, resulting in a rapid decrease in vibration.

Once again, this system can be represented by a block diagram like the one shown in Figure 4. The "cutting stiffness" is the stiffness of the material coating. The machine dynamics are the relative dynamics between the shaft and shell. Regenerative feedback occurred due to the time delay associated with one shaft revolution. While an accurate representation of the dynamics would be exceedingly difficult to develop due to the number of scrapers and the complexity of the coating material, the feedback paradigm provides helpful directional information.

Once the instability was identified and understood, the following solutions were considered:

1. Eliminating the material coating.
2. Installing a larger diameter shaft.
3. Changing the scraper geometry and/or modifying the properties of the coating.
4. Installing tuned dampers on the shaft.

Eliminating the material coating on the shell interior proved to be impractical. If this could have been accomplished, the feedback tie between the structural vibration and the coating thickness variation would have been broken. However, testing revealed that the material coating was required to adequately machine the final product.

Installing a larger shaft was seriously pursued, but found to involve many process risks. Calculations indicated that the shaft diameter would have to double to gain adequate protection against the instability. The resulting increase in shaft weight, and the effect on product flow within the mixer, made this option unattractive. Further, there was no guarantee that the instability would be eliminated.

Changing the scraping tools involved too many process risks

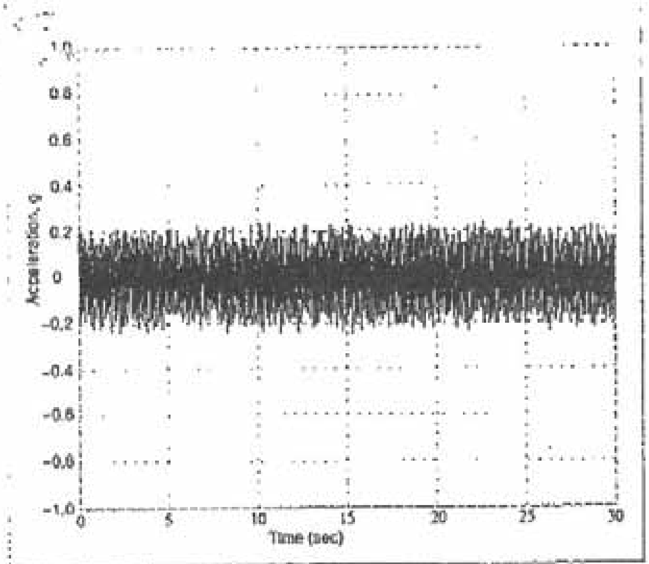

Figure 12. Time history measured on the mixer frame following installation of tuned dampers.

and unknowns. The objective of this investigation was to reduce the "cutting stiffness" by choosing more efficient scrapers.

Option No. 4, installing tuned dampers on the shaft, was the only practical solution path consistent with process constraints. Tuned dampers increase the dynamic stiffness of the shaft and shell, resulting in less vibration for a given input force. This solution was considered low risk in that the dampers could be removed if they provided little or no benefit.

Tuned dampers were installed on the mixer shaft, which increased the shaft stiffness by a factor of four. The result was that vibration was reduced by an order of magnitude and the instability was eliminated for 95% of process conditions.

Figure 12 shows a time history recorded after the dampers were installed. Note that the character of the vibration is substantially different than that shown in Figure 10. Once the instability was eliminated, the vibration buildup (due to positive reinforcement of the pattern in the material coating) no longer occurred. The vibration was much more uniform.

Summary

Mechanical instabilities, while relatively rare, do arise in process industries. Instabilities and resonance are similar in that both are associated with natural frequencies; otherwise, they are vastly different phenomena. Instabilities develop spontaneously due to the presence of a feedback mechanism between structural vibration and the oscillation of a process variable. Several methods are available for eliminating instabilities including: (1) eliminating the feedback path; (2) reducing the "cutting stiffness;" and (3) increasing the dynamic stiffness of the structure. Increasing the dynamic stiffness is often the most practical alternative. Dramatic reductions in vibration amplitude are possible once the instability is eliminated.

Acknowledgements

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